

Unit-2 (i) Antenna Arrays

Antenna Arrays:-

Antenna Array is one of the common method to combining the radiations from group of similar antenna.

Total electric field from an array is equal to sum of individual antenna electric fields.

Antenna arrays are mainly used to transmit radiation pattern at a great distance with maximum electric field.

Linear Antenna Arrays:-

An Antenna array is said to be linear, if the individual antennas of the array equally space along the st. line.

Uniform linear Array:-

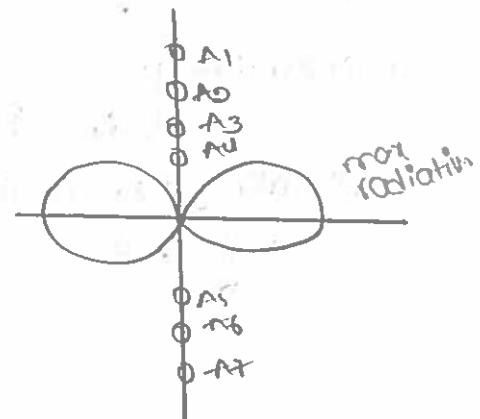
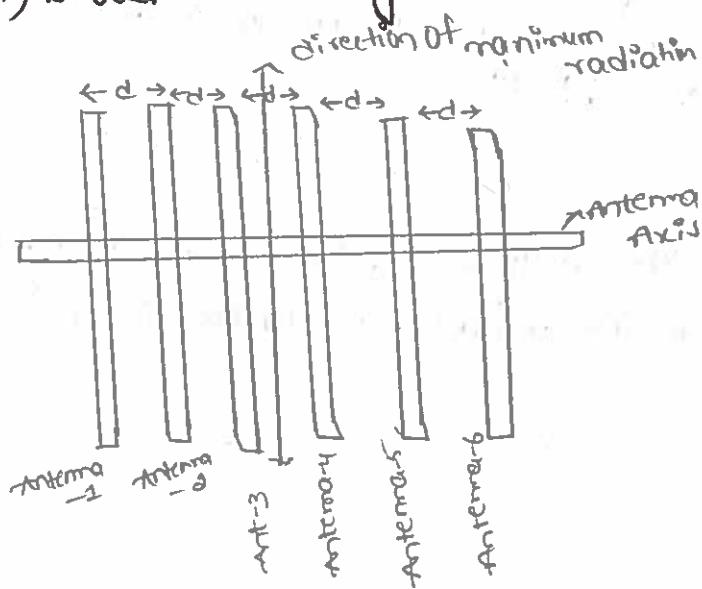
An antenna is said to be uniform linear array, if the individual linear antennas fed with current of equal magnitude and phase.

Types of Antenna Arrays:-

Depending on radiation pattern & arrangement on individual antenna. Antenna arrays are 3 types.

- * Broad side array
- * End fire array
- * Collinear array

i) Broad side array:-



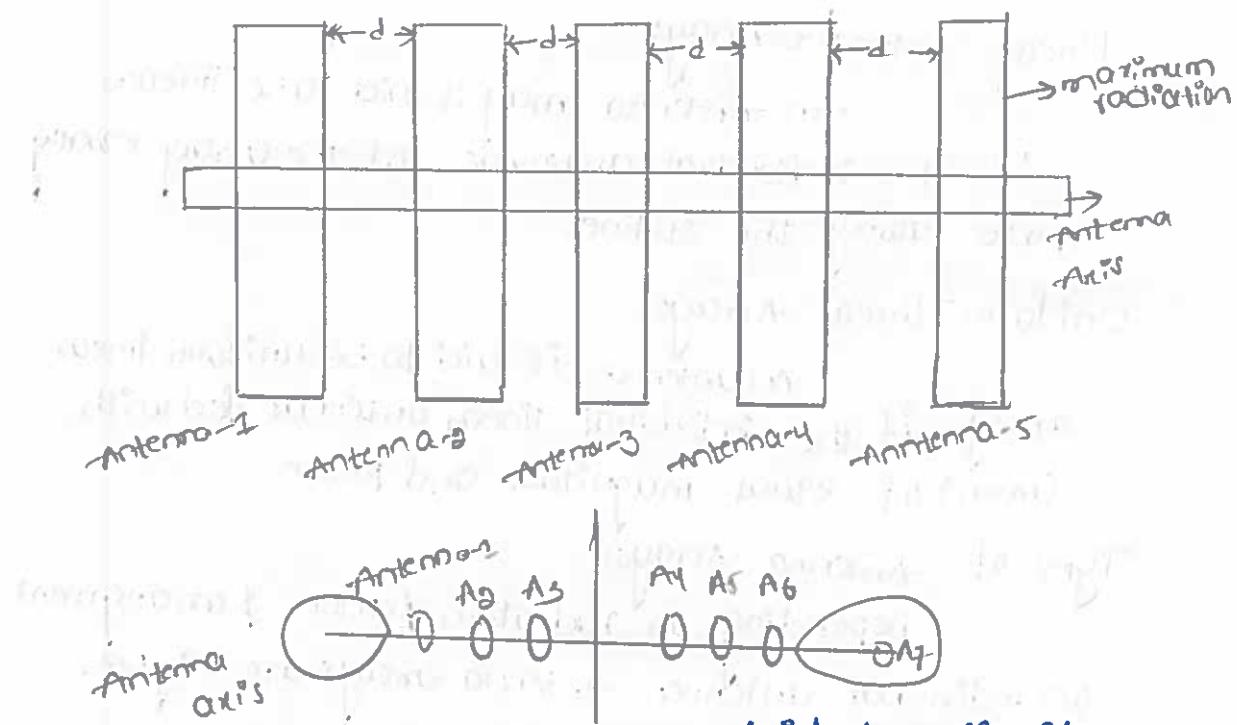
In the broadside array, individual antennas are equally spaced along the line & each element is fed with current of equal magnitude & same phase.

Due to this arrangement the radiation pattern produced in the broadside direction.

i.e., far to the line of antenna array axis.

Hence, in the broadside antenna array the radiation produced which is in bidirectional.

a) End fire array:-

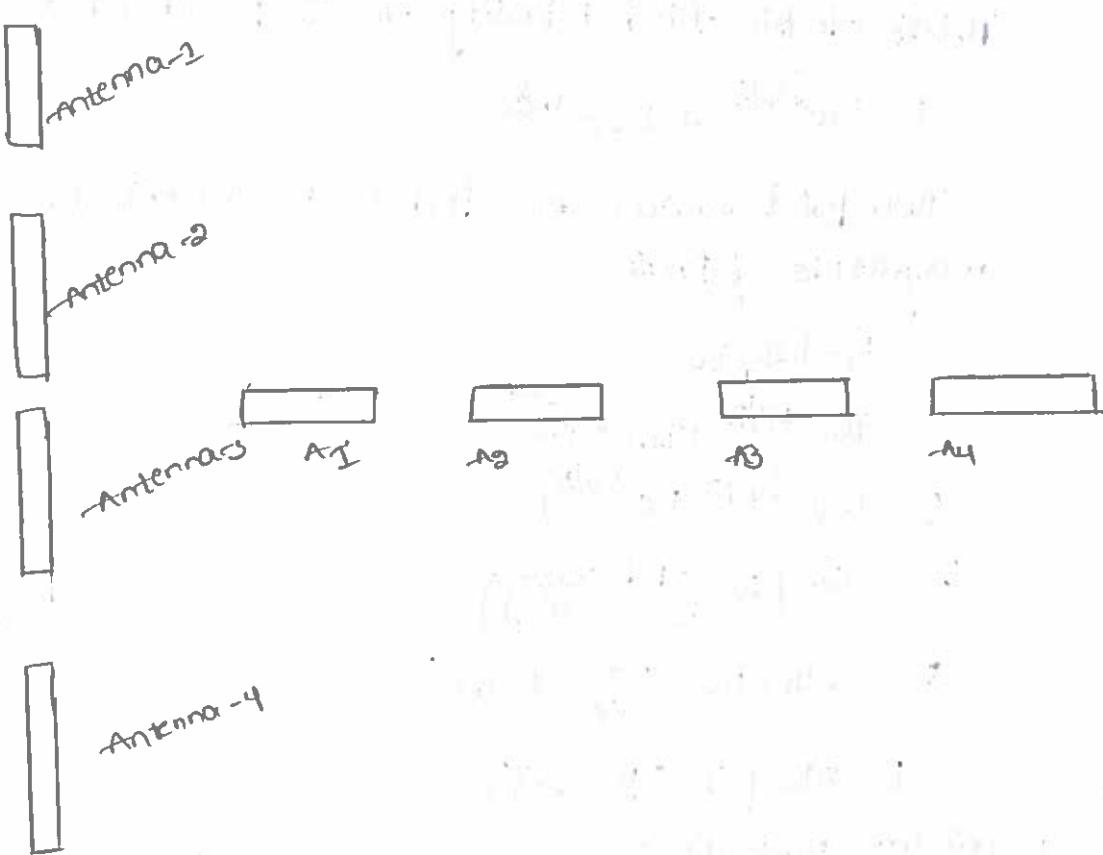


In the end fire array antenna, individual antennas are equally spaced along a line & each element is fed with current of equal magnitude & their phases are varies, progressively along the line (180° out of phase).

Due to this arrangement maximum radiation produced along end fire direction

Collinear Array:-

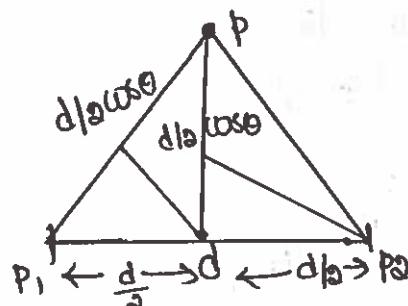
In collinear array the antennas are arranged coaxially i.e., antennas are mounted end to end in a single line.



Arrays of two point sources:-

Arrays of two point sources with equal magnitude

& phases-



Consider, two point sources P_1 & P_2 separated by a distance 'd' path difference from P_1 to P is $d/\sin\theta$. and path difference from P_2 to P also $d/\sin\theta$.

Total path difference is given by,

$$\text{path difference} = \frac{d}{\sin\theta} + \frac{d}{\sin\theta}$$

$$= d \cos\theta \text{ in meters}$$

$$= \frac{d}{\lambda} \cos\theta \text{ wave length}$$

Phase angle $= 2\pi(\text{path difference})$

$$= 2\pi \left(\frac{d}{\lambda} \cos\theta \right)$$

$$\Psi = \beta d \cos\theta \quad \left[\because \beta = \frac{2\pi}{\lambda} \right]$$

Total electric field intensity due to point & point-d

$$E = E_1 e^{-j\psi/2} + E_2 e^{j\psi/2}$$

Two point sources are fed with current of equal magnitude & phase.

$$E_1 = E_2 = E_0$$

$$E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E = E_0 (e^{-j\psi/2} + e^{j\psi/2})$$

$$E = 2E_0 (\cos(\frac{\beta d \cos\theta}{\lambda}))$$

$$E = 2E_0 (\cos(\frac{2\pi}{\lambda} \cos\theta))$$

$$E = 2E_0 (\cos(\frac{\pi}{\lambda} \cos\theta))$$

maximum Radiation:-

$$\cos(\frac{\pi}{\lambda} \cos\theta) = \pm 1$$

$$\cos(\frac{\pi}{\lambda} \cos\theta) = \cos 0^\circ$$

$$\frac{\pi}{\lambda} \cos\theta = 0$$

$$\cos\theta = 0$$

$$\cos\theta = \cos \pi/2$$

$$\boxed{\theta = \pi/2} \quad 87^\circ$$

Minimum radiation :-

$$\cos(\frac{\pi}{\lambda} \cos\theta) = 0$$

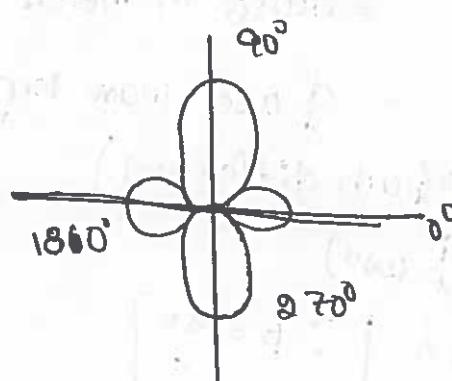
$$\cos(\frac{\pi}{\lambda} \cos\theta) = \cos \pi/2$$

$$\frac{\pi}{\lambda} \cos\theta = \pi/2$$

$$\cos\theta = \pm 1$$

$$\cos\theta = \cos 0^\circ$$

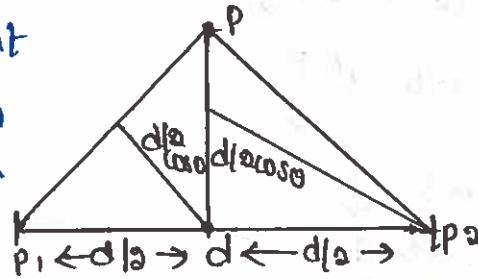
$$\theta = 0^\circ ; 180^\circ \& 90^\circ$$



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Arrays of two point sources with equal magnitude & opposite phase.

In this case 1, point source is fed with 180° out of phase.



Consider two point sources $P_1 \& P_2$ separated by a distance d : path difference from P_1 to P_2 is $\frac{d}{2} \cos\theta$. and path difference from P_2 to P_1 is $\frac{d}{2} \cos\theta$.

Total path difference is given by,

$$\text{Path difference} = \frac{d}{2} \cos\theta + \frac{d}{2} \cos\theta$$

$$= d \cos\theta \text{ in meters}$$

$$= \frac{d}{\lambda} \cos\theta \cdot \text{wave length}$$

$$\text{Phase angle} = 2\pi (\text{path difference})$$

$$= 2\pi \frac{d}{\lambda} \cos\theta$$

$$\Psi_B d \cos\theta \quad \left[\because \beta = \frac{2\pi}{\lambda} \right]$$

Total electric field intensity due to $P_1 \& P_2$

$$E = E_1 e^{i\Psi_B} + E_2 e^{-i\Psi_B}$$

TWO point sources are fed with current of equal magnitude & opposite phase.

$$E_1 = E_0$$

$$E_2 = -E_0$$

$$E = E_0 e^{i\Psi_B} - E_0 e^{-i\Psi_B}$$

$$E = E_0 (e^{i\Psi_B} - e^{-i\Psi_B})$$

$$E = 2E_0 (\sin(\frac{\Psi_B}{2}))$$

$$E = 2E_0 \left(\sin \left(\frac{\beta d \cos\theta}{2} \right) \right)$$

$$E = 2E_0 \left(\sin \left(\frac{2\pi}{\lambda} \cdot \frac{d}{2} \cos\theta \right) \right)$$

$$E = 2E_0 \left(\sin \left(\frac{\pi}{\lambda} d \cos\theta \right) \right)$$

maximum Radiation:-

$$\sin\left(\frac{\pi}{2} \cos\theta\right) = \pm 1$$

$$\sin\left(\frac{\pi}{2} \cos\theta\right) = \sin\frac{\pi}{2}$$

$$\frac{\pi}{2} \cos\theta = \frac{\sin\frac{\pi}{2}}{\sin}$$

$$\frac{\pi}{2} \cos\theta = \frac{\pi}{2}$$

$$\cos\theta = 1$$

$$\cos\theta = \cos 0^\circ$$

$$\theta = 0^\circ, 180^\circ$$

minimum Radiation:-

$$\sin\left(\frac{\pi}{2} \cos\theta\right) = 0$$

$$\sin\left(\frac{\pi}{2} \cos\theta\right) = \sin 0^\circ$$

$$\frac{\pi}{2} \cos\theta = 0$$

$$\cos\theta = 0^\circ$$

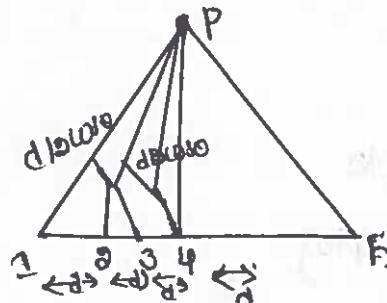
$$\Rightarrow \cos\theta = \cos\frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{2}; 270^\circ.$$

Case 3:- Linear array with n -isotropic point sources with equal magnitude & phase.

Consider, ' n ' point sources with equally space & they are fed with equal magnitude & phase then total electric field at a point P due to n -point source is given by,

$$E = E_0 e^{j\psi(1)} + E_0 e^{-j\psi(1)}$$



$$E = E_0 e^{j\psi(1)} + E_0 e^{-j\psi(1)}$$

$$E = E_0 (e^{j\psi(1)} + e^{-j\psi(1)})$$

$$E_t = E_0 e^{j\psi(0)} + E_0 e^{j\psi(1)} + E_0 e^{j\psi(2)} + \dots + E_0 e^{j\psi(n-1)}$$

$$E_t = E_0 (1 + e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \dots + e^{(n-1)j\psi}) - ①$$

Multiply eqn ① with $e^{j\psi}$

$$E_t - e^{j\psi} E_t = E_0 (1 - e^{jn\psi})$$

$$E_t = \frac{E_0 (1 - e^{jn\psi})}{(1 - e^{j\psi})}$$

$$E_t = E_0 \frac{(1 - e^{jn\psi}) e^{jn\psi/2}, e^{jn\psi/2}}{(1 - e^{j\psi}) e^{jn\psi/2}}$$

$$E_t = E_0 \frac{e^{jn\psi/2} (e^{-jn\psi/2} - e^{jn\psi/2})}{e^{jn\psi/2} (e^{-jn\psi/2} - e^{jn\psi/2})}$$

$$E_t = E_0 + \frac{e [j n \psi / 2 - e^{-j \psi / 2}]}{+ e [e^{j \psi / 2} - e^{-j \psi / 2}]} e^{jn\psi/2} - e^{jn\psi/2}$$

$$E_t = E_0 \frac{\sin(\frac{n\psi}{2})}{\sin(\frac{\psi}{2})} e^{j\frac{\psi}{2}(n-1)}$$

$$E_t = E_0 \frac{\sin(\frac{n\psi}{2})}{\sin(\frac{\psi}{2})} e^j$$

$$E_t = E_0 \frac{\sin(n\frac{\psi}{2})}{\sin(\frac{\psi}{2})} (\cos\psi + j \sin\psi)$$

Maximum Radiation of minor lobe:-

$$\sin(\frac{n\psi}{2}) = \pm 1$$

$$\sin(\frac{n\psi}{2}) = \sin(\pm(2N+1)\frac{\pi}{2})$$

$$\frac{n\psi}{2} = \pm(2N+1)\frac{\pi}{2}$$

$$n\psi = \pm(2N+1)\pi$$

$$\psi = (2N+1)\frac{\pi}{n} \quad [\because \psi = \beta d \cos\theta + \alpha]$$

$$(\beta \cos(\theta_{\text{max}})_{\text{minor side}}) = \pm(2N+1)\frac{\pi}{n}$$

$$(\beta \cos(\theta_{\text{max}})_{\text{minor side}}) = \pm(2N+1)\frac{\pi}{n}$$

$$\beta \cos(\theta_{\text{max}})_{\text{minor side}} = \pm(2N+1)\frac{\pi}{n} - \alpha$$

'id' represents whether the broad side (α) end fire side.

$$\cos(\theta_{\text{min}}) \text{ minor} = \frac{1}{pd} \left(\pm (2N+1) \frac{\pi}{n} - \alpha \right)$$

$$(\theta_{\text{min}}) \text{ minor} = \cos^{-1} \left(\frac{1}{pd} \pm (2N+1) \frac{\pi}{n} - \alpha \right)$$

minimum radiation for minor lobe-

$$\sin \left(\frac{n\psi}{8} \right) = 0$$

$$\sin \left(\frac{n\psi}{8} \right) = \pm \sin N\pi$$

$$\frac{n\psi}{8} = \pm N\pi$$

$$\frac{n\psi}{8} = \pm \frac{\pi N}{n}$$

$$n\psi = \pm 8\pi N \Rightarrow \psi = \pm \frac{8N\pi}{n}$$

$$(pd \cos(\theta_{\text{min}}) \text{ minor} + \alpha) = \pm \frac{8N\pi}{n}$$

$$pd \cos(\theta_{\text{min}}) \text{ minor} = \pm \frac{8N\pi}{n} - \alpha$$

$$\cos(\theta_{\text{min}}) \text{ minor} = \frac{1}{pd} \left(\pm \frac{8N\pi}{n} - \alpha \right)$$

$$(\theta_{\text{min}}) \text{ minor} = \cos^{-1} \left(\frac{1}{pd} \pm \frac{8N\pi}{n} - \alpha \right)$$

for, broad side array;

$$\psi = pd \cos(\theta + \alpha)$$

for, broad side array maximum radiation produced
in the direction of 90° (or) 270°

$$\text{so, } \theta = 90^\circ \text{ (or) } 270^\circ$$

consider, phase angle, $\psi = 0$

$$\theta = pd \cos 90^\circ + \alpha$$

$$\boxed{\alpha = 0}$$

$$(\theta_{\text{min}}) \text{ minor} = \cos^{-1} \left(\frac{1}{pd} \pm (2N+1) \frac{\pi}{n} - \alpha \right)$$

$$= \cos^{-1} \left(\frac{1}{pd} \pm (2N+1) \frac{\pi}{n} \right)$$

$$= \cos^{-1} \left(\frac{1}{\frac{8N\pi}{n} d} \pm (2N+1) \frac{\pi}{n} \right)$$

$$= \cos^{-1} \left(\frac{1 \pm (2N+1)}{\frac{8N\pi}{n} d} \right)$$

$$(\theta_{\text{min}}) \text{ minor} = \cos^{-1} \left(\frac{1 \pm (2N+1)}{\frac{8N\pi}{n} d} \right)$$

$$= \cos^{-1} \left(\frac{1}{\beta d} \pm \frac{2n\pi}{N} \right)$$

$$= \cos^{-1} \left(\frac{1}{\frac{\lambda}{2\pi} d} \pm \frac{2n\pi}{N} \right)$$

$$= \cos^{-1} \left(\frac{1 \pm \frac{2n\pi}{N}}{\frac{\lambda}{2\pi} d} \right)$$

For end fire side array

$$\Psi = \beta d \cos \theta + \alpha$$

For end fire side array maximum radiation produced in the direction of 0° ($\theta = 0^\circ$) 180° $\theta = 180^\circ$

Consider, phase angle, $\Psi = 0$

$$\theta = \beta d \cos \theta + \alpha$$

$$\theta = \beta d + \alpha$$

$$\alpha = -\beta d$$

$$\begin{aligned} (\theta_{\max})_{\min} &= \cos^{-1} \left(\frac{1}{\beta d} \pm \frac{(2N+1)\pi}{N} + \beta d \right) \\ &= \cos^{-1} \left(\frac{1}{\frac{\lambda}{2\pi} d} \pm \frac{(2N+1)\pi}{N} + \beta d \right) \\ &= \cos^{-1} \left(\frac{1 \pm \frac{(2N+1)\pi}{N}}{\frac{\lambda}{2\pi} d} + \beta d \right) \\ &= \cos^{-1} \left(\frac{1 \pm \frac{(2N+1)}{N}}{\frac{\lambda}{2\pi} d} + \beta d \right) \end{aligned}$$

$$\begin{aligned} (\theta_{\min})_{\max} &= \cos^{-1} \left(\frac{1}{\beta d} \pm \frac{2\pi n}{N} - \alpha \right) \\ &= \cos^{-1} \left(\frac{1}{\frac{\lambda}{2\pi} d} \pm \frac{2\pi n}{N} + \beta d \right) \\ &= \cos^{-1} \left(\frac{1}{\frac{\lambda}{2\pi} d} \pm \frac{2\pi n}{N} + \beta d \right) \end{aligned}$$

Binomial Array:-

In previous notes, we discuss about linear arrays of n -isotropic sources of equal magnitudes but arrays of non-uniform amplitudes also possible using binomial array.

In this array, the amplitudes of the each antenna arranged according to coefficients of binomial series.

$$(a+b)^{n-1} = a^{n-1} + \frac{(n-1)}{1!} a^{n-2} b + \frac{(n-1)(n-2)}{2!} a^{n-3} b^2 + \frac{(n-1)(n-2)(n-3)}{3!} a^{n-4} b^3 + \dots$$

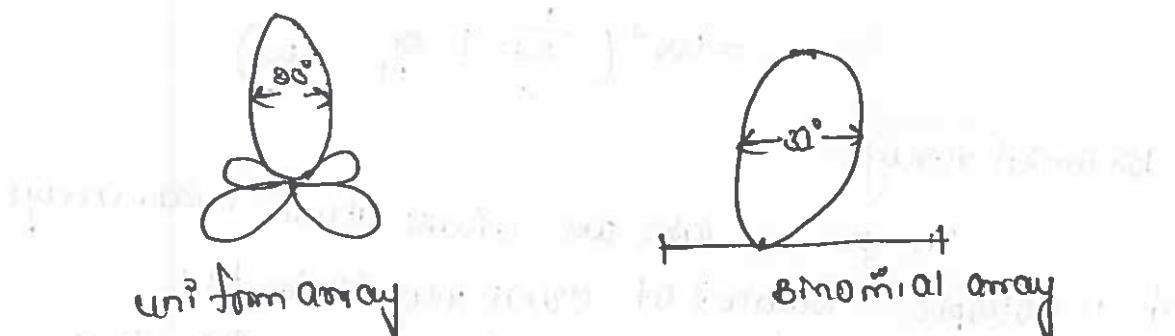
In uniform linear array the array length is increased to increase the directivity but, array line increased minor lobe also increases.

But, certain applications it is desirable that secondary lobe should be eliminated.

To eliminate the secondary lobes by using binomial array. The coefficients of each antenna in an array obtained by using Pascal triangle.

$n=1$	1
$n=2$	1 1
$n=3$	1 2 1
$n=4$	1 3 3 1
$n=5$	1 4 6 4 1
$n=6$	1 5 10 10 5 1

Consider, $n=5$ λ spacing between antenna 10° then HPBW of binomial & uniform linear array are 25° , 82° respectively. That means in uniform linear array minor lobes are appear but major lobes are sharp & narrow, where as in binomial array width of the major lobe is wide but, no minor lobes.



(ii) Antenna Measurements

Different types of Regions:-

There are 2 types of regions around the antenna based on distance.

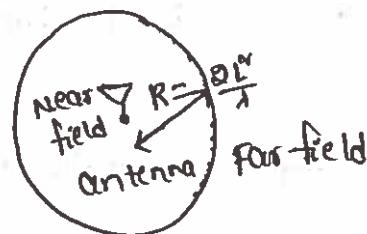
- 1) Near field Region
- 2) Far field Region

* The Region near to the antenna is called "Near field region".

* The Region at a large distance from antenna is called "Far field region".

$$\text{minimum radius of FFR is, } R = \frac{2L^2}{\lambda}$$

where, 'L' is maximum dimension of antenna.



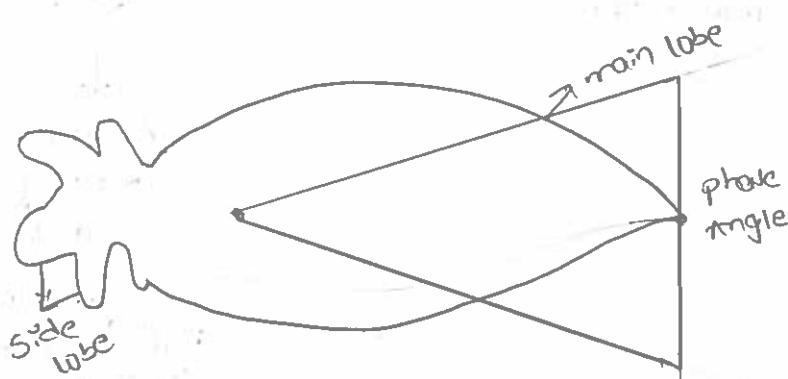
Sources of Errors in Antenna Measurements:-

Any measured quantity has a margin of error in antenna measurements. There are 2 types of errors.

i) Phase error & Amplitude error due to finite measurement distance.

ii) Error due to reflections.

Phase error & Amplitude error due to finite measurement distance:-



Let us assume that AUT (Antenna under Test) is a receiving antenna which is receiving the wave from transmitting antenna, if the measurement distance is too small the fields received by different parts of AUT antenna will not be in phase & there will be phase errors.

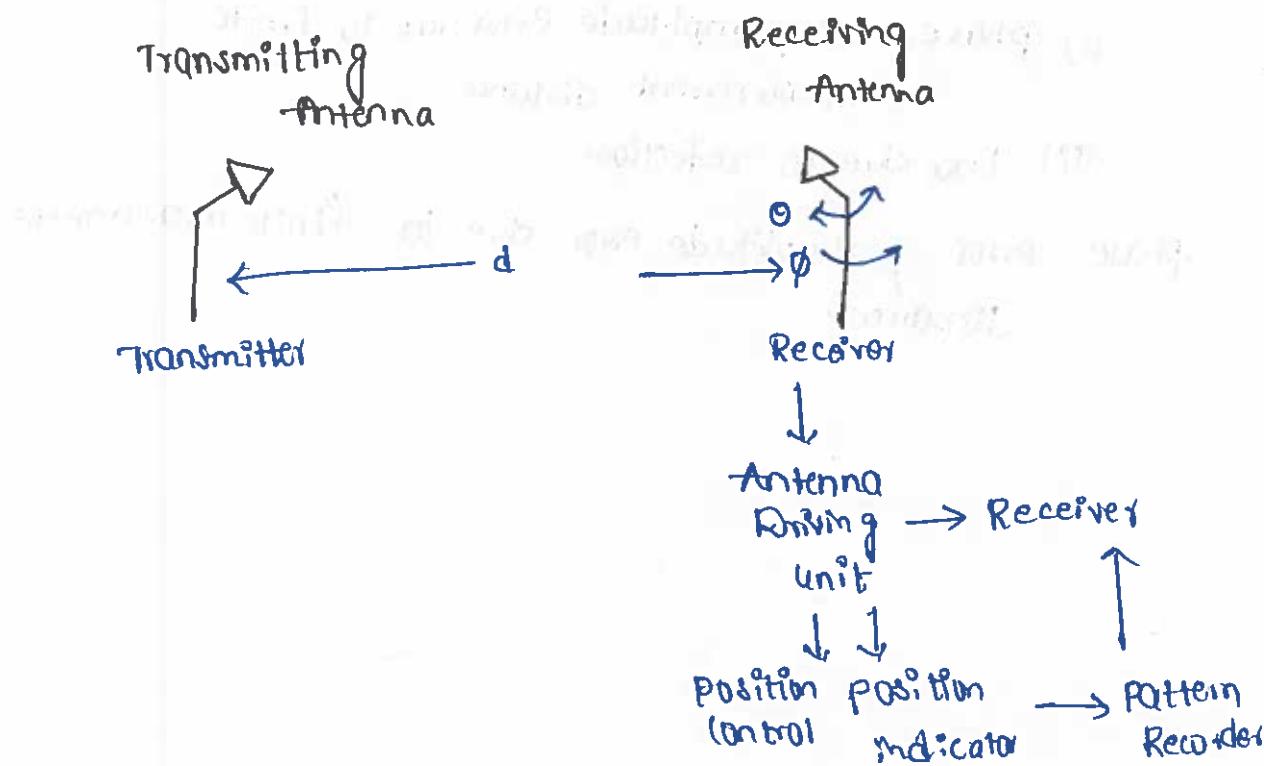
Due to this phase error gain becomes smaller & lobes are higher compared with transmitting radiation pattern for measuring antennas having moderate side lobes minimum $\frac{2\pi}{\lambda}$ distance is required.

Errors due to Reflections-

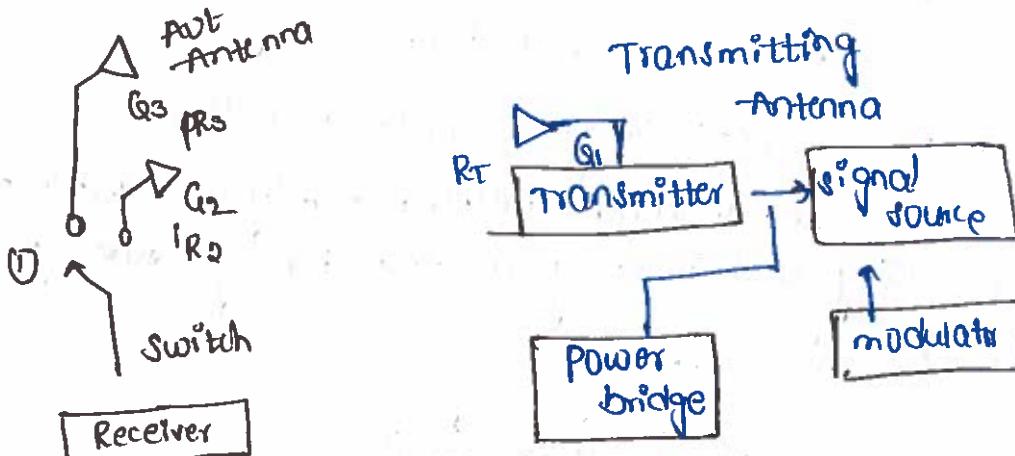
Reflection from surrounding produced field variations due to interference of direct wave and reflected waves. Even small reflected waves cause large measurement errors because, the fields of direct & reflected waves are added.

Antenna Radiation pattern measurement-

Radiation pattern of transmitting antenna is defined as field strength (or) power strength of at a fixed point from the antenna as a function of direction.



- The radiation pattern of antenna is 3D it needs measurement of field strength in θ, ϕ directions & record the receiver power. antenna driving unit is used to drive the antenna in different directions using positional control & positional indicator.
- positional control to control the antenna direction, position indicator records different θ, ϕ values.



In comparison, method we are considering is Antennas

- * Transmitting antenna
- * Reference antenna
- * Antenna under test antenna

The distance b/w test antenna and Receiving antenna should be greater than $\frac{\lambda}{4\pi R}$, that is, for field distance. The transmitted power ' P_t ' which is feeding with signal source.

Friis formulae

$$P_R = P_T G_T G_R$$

$$P_{R2} = P_T G_1 G_2 \left(\frac{1}{4\pi R} \right)^2 - ①$$

consider, switch is at position -1

$$P_{R3} = P_T G_1 G_3 \left(\frac{1}{4\pi R} \right)^2 - ②$$

$\frac{\text{eqn } ①}{\text{eqn } ②}$

$$\frac{P_{R2}}{P_{R3}} = \frac{P_T G_1 G_2 \left(\frac{1}{4\pi R} \right)^2}{P_T G_1 G_3 \left(\frac{1}{4\pi R} \right)^2}$$

$$\frac{P_{R2}}{P_{R3}} \approx \frac{G_2}{G_3}$$

$$G_3 = G_2 \left(\frac{P_{R3}}{P_{R2}} \right)$$

By using above formulae, we find unknown gain of AUT antenna.

- Antenna gain measurement using absolute method:-

In absolute method there are 2 types of dependings on number of antennas.

- * Two antenna method.

- * Three antenna method.

Antenna measurement using 2-Antenna method:-

2-Antenna method mainly used when both transmitting and receiving antenna having same gain.

By using free formulae,

$$P_R = P_T G_1 G_2 \left(\frac{\lambda}{4\pi R} \right)^2$$

$$G_1 G_2 = \left(\frac{P_R}{P_T} \right) \left(\frac{4\pi R}{\lambda} \right)^2$$

Multiply by \log on both sides.

$$10 \log (G_1 G_2) = 10 \log \left(\frac{P_R}{P_T} \right) \left(\frac{4\pi R}{\lambda} \right)^2$$

$$10 \log (G_1) + 10 \log (G_2)$$

$$= 10 \log \left(\frac{P_R}{P_T} \right) + 10 \log \left(\frac{4\pi R}{\lambda} \right)$$

$$G_1 = G_2 = G$$

$$= 2(10 \log(G)) = 10 \log \left(\frac{P_R}{P_T} \right) + 20 \log \left(\frac{4\pi R}{\lambda} \right)$$

$$(G)_{dB} = \frac{1}{2} \left(10 \log \left(\frac{P_R}{P_T} \right) + 20 \log \left(\frac{4\pi R}{\lambda} \right) \right)$$

Gain measurement using 3-Antenna method

Consider, 3-antennas having gain G_1 , G_2 & G_3 .

Apply, free formulae to antenna 1 & antenna 3.

$$P_{R2} = P_T G_1 G_2 \left(\frac{\lambda}{4\pi R} \right)^2$$

Apply Friis formulae for antenna-1 & antenna-3

$$P_{R2} = P_{T1} G_1 G_2 \left(\frac{1}{4\pi R} \right)^2$$

$$P_{R3} = P_{T1} G_1 G_3 \left(\frac{1}{4\pi R} \right)^2$$

$$G_1 G_2 = \left(\frac{P_{R2}}{P_{T1}} \right) \left(\frac{4\pi R}{\lambda} \right)^2$$

Multiplying by log on both sides.

$$10 \log G_1 + 10 \log G_2 = 10 \log \left(\frac{P_{R2}}{P_{T1}} \right) + 20 \log \left(\frac{4\pi R}{\lambda} \right)$$

$$(G_1) dB + (G_2) dB = 10 \log \left(\frac{P_{R2}}{P_{T1}} \right) + 20 \log \left(\frac{4\pi R}{\lambda} \right) - ①$$

Similarly, antenna 2 & 3 antennas

$$P_{R3} = P_{T1} G_2 G_3 \left(\frac{1}{4\pi R} \right)^2$$

$$G_2 G_3 = \left[\frac{P_{R3}}{P_{T1}} \right] \left(\frac{4\pi R}{\lambda} \right)^2$$

$$(G_2) dB + (G_3) dB = 10 \log \left(\frac{P_{R3}}{P_{T1}} \right) + 20 \log \left(\frac{4\pi R}{\lambda} \right) - ②$$

Similarly, antenna 1 & 3

$$P_{R3} = P_{T1} G_2 G_3 \left(\frac{1}{4\pi R} \right)^2$$

$$G_2 G_3 = \left(\frac{P_{R3}}{P_{T1}} \right) \left(\frac{4\pi R}{\lambda} \right)^2$$

$$(G_2) dB + (G_3) dB = 10 \log \left(\frac{P_{R3}}{P_{T1}} \right) + 20 \log \left(\frac{4\pi R}{\lambda} \right) - ③$$

Eqs ① & ② & ③

$$G_2(dB) - G_3(dB) = 10 \log \left(\frac{P_{R2}}{P_{T1}} \right) - 10 \log \left(\frac{P_{R3}}{P_{T1}} \right)$$

$$\left[\because \log \frac{a}{b} = \log a - \log b \right]$$

$$G_2(dB) - G_3(dB) = 10 \log \left(\frac{P_{R2}}{P_{R3}} \right) - ④$$

Eqn ③ & eqn ④

$$G_2(dB) = 10 \log \left(\frac{P_{R2}}{P_{R3}} \right) + 10 \log \left(\frac{P_{R3}}{P_{T1}} \right)$$

$$G_2(dB) = \frac{1}{2} \left[\log \left(\frac{P_{R2}}{P_{T1}} \right) + 20 \log \left(\frac{4\pi R}{\lambda} \right) \right]$$

connected by a bridge with velocity v_0 along
 (\hat{m}^1) and v_1 along
 (\hat{m}^2)

$\left(\hat{m}^1\right) \text{ and } \left(\hat{m}^2\right) \text{ are}$

coupled by a pair of constraints

$\left(\hat{m}^1\right) \text{ and } \left(\hat{m}^2\right) \text{ have initial velocities}$

$\left(\hat{m}^1\right) \text{ initial } v_0 \text{ and } \left(\hat{m}^2\right) \text{ initial } v_1$

and they are coupled by

$\left(\hat{m}^1\right) \text{ and } \left(\hat{m}^2\right) \text{ are}$

$\left(\hat{m}^1\right) \text{ and } \left(\hat{m}^2\right) \text{ have initial velocities}$

and they are coupled by

$\left(\hat{m}^1\right) \text{ and } \left(\hat{m}^2\right) \text{ are}$

$\left(\hat{m}^1\right) \text{ and } \left(\hat{m}^2\right) \text{ are}$

$\left(\hat{m}^1\right) \text{ and } \left(\hat{m}^2\right) \text{ have initial velocities}$

and they are coupled by

$\left(\hat{m}^1\right) \text{ and } \left(\hat{m}^2\right) \text{ have initial velocities}$

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